A switched LPV observer for actuator fault estimation

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Abstract: In this paper, an actuator fault estimation problem is tackled, based on the LPV approach. First, the actuator fault is modeled in the form of a multiplicative fault by using a constant coefficient representing the loss of efficiency of the actuator's power α. Therefore, the estimation of a time varying faulty actuator can be transformed into the estimation of a constant coefficient α. Then, the faulty system is rewritten in the form of a switched LPV system. The coefficient α and the system states are estimated using an extended switched observer. The stability and performance of the observer is ensured considering a switched time-dependent Lyapunov function. The observer gains are derived based on LMI solutions for polytopic switched systems. Some simulation results are presented that show the effectiveness of this approach.

Keywords: Multiplicative fault, fault estimation, switched extended observer, *LPV* system

1. INTRODUCTION

Fault diagnosis plays an important role in the control and supervision of the industrial systems in order to enhance the safety, reliability and to reduce the loss of productivity. Nowadays, it has been attracted more and more attention from the researchers. In fact, fault estimation is a key step in Fault Detection and Isolation (FDI). More precisely, a FDI strategy can be model-based or data-based and is used to detect, isolate as well as estimate the faults. During the last decades, FDI modules, based on the analytical redundancy for fault estimation (e.g in Zhang and Jiang (2008)), have received a lot of attention by many researchers. In the literature, there exist many different approaches to estimate a fault occurring either on the actuator or on the sensor. Let us mention the classical methods, based on the parity space theory (see in Gertler (1997)) to generate the residues and approximate the fault or the bank of observers approach (see Varrier (2013)) as well as by sliding mode observers (Edwards et al. (2000)). Recently, a new approach (see in Shi and Patton (2014)) considered the fault element as a state of the augmented system and designed an extended observer to estimate at the same time the state and the fault of the system. However, it is limited to constant faults $\dot{f}(t) = 0$. Then, Zhang et al. (2008) presented a method allowing to evaluate the time-varying fault by using a fast adaptive fault estimation (FAFE) methodology based on an adaptive observer. But therein, the authors solved the problem with a regular Linear Time Invariant (LTI) system without considering the disturbances. Next, Rodrigues et al. (2014) proposed an adaptive polytopic observer for time-varying fault estimation in spite of the disturbances for a class of descriptor Linear Parameter Varying (LPV) systems. Most of these works considered the fault as an additive one or transformed a multiplicative fault into an additive one.

The main purpose of this paper is to propose a methodology to estimate multiplicative faults for the actuators. An actuator time-varying fault is considered in the form of actuator power loss. Then, a constant "fault" coefficient is used to model the power loss. In this way, a constant coefficient should be estimated rather than the time-varying fault signal. The paper contribution are twofold:

- First, the faulty system is modeled in the form of an LPV system by considering the control input as a scheduling parameter. Then, the LPV system is rewritten to a switched LPV system.
- Second, an LPV extended switched observer is designed to estimate both the "faulty" coefficient α and the system states. Two case studies are considered: the system without and with input disturbance. Some conditions on the decay rate of observer are added in order to increase the convergence's speed of estimation.

The paper is organized as follows: the next section recalls some preliminaries on the stability of the switched systems. Section 3 presents the problem statement. Section 4 gives a full description for an actuator multiplicative fault estimation based on the switched observer. Section 5 shows some simulation results of the proposed method using an academic example. Finally, some conclusions are drawn in the section 6.

2. PRELIMINARIES

This section is devoted to recall some results on the stability analysis for the continuous time, switched system by using the multiple Lyapunov function. Let us consider the following switched linear system:

$$\dot{x}(t) = A_{\sigma(t)}x(t), \qquad x(0) = x_0, \tag{1}$$

where $\sigma(t)$ is the switching signal and $A_{\sigma(t)} \in \{A_1, ..., A_M\}$, $A_i \in R^{n\times n}, i = 1...M$. Obviously, this model imposes discontinuity in $A_{\sigma(t)}$ since this matrix jumps instantaneously from A_i to A_j for $i \neq j$.

Now, the stability of the switched system (1) is guaranteed if there exists a family of symmetric and positive Lyapunov matrices $\{P_1, ...P_M\}$, each one associated to the correspondent A_i such that the Lyapunov function $V_{\sigma(t)}(x(t))$ is non increasing for all $t \geq 0$ (Liberzon (2003)). Indeed we need to ensure that the Lyapunov function is non increasing at the switching instants ($V(x(t_k) \leq V(x(t_k^-))$ with t_k is the switching instant). This condition is conservative and it is replaced by a weaker condition that the sequence $V(x(t_k))$ is decreasing for $t_k, k = 0, ..., \infty$ is switching instant (Geromel and Colaneri (2006)) i.e if t_k and t_{k+1} are successive switching times such that $\sigma(t_k) = i$ and $\sigma(t_{k+1}) = j$ then $V_j(x(t_{k+1})) \leq V_i(x(t_k))$. To ensure the global stability under slow switching, the concept of "dwell time τ_d" is considered imposing that the interval between 2 successive switching instants satisfies $t_{h+1} - t_h \geq \tau_d$, for all switching times. However, in the design step, it is not easy to handle these conditions in terms of convex optimization (and therefore as LMIs), specially for systems with uncertainties or polytopic systems. An interesting solution has been recently proposed by Allerhand et al. (2011), using a time-dependent parametrized Lyapunov function: $V_\sigma(t) = x(t)' P_\sigma(t) x(t)$, where $P_{\sigma(t)}(t)$ is the Lyapunov matrix and is given by:

$$P_{\sigma(t)}(t) = \begin{cases} P_{i,k} + (P_{i,k+1} - P_{i,k})\dfrac{t - \tau_{s,k}}{T/K} := \hat{P}_{i,k} & if \quad t \in [\tau_{s,k},\ \tau_{s,k+1}) \\ P_{i,K} & if \quad t \in [\tau_{s,K},\ \tau_{s+1,0}) \\ P_{i_0,K} & if \quad t \in [0, \tau_1) \end{cases} \quad (2)$$

where $i = 1, ...N$ with N is number of subsystems, $i_0 = \sigma(0)$. $\tau_1, \tau_2, ...$ are the switching instants. T is the dwell time satisfying $\tau_{s+1} - \tau_s \geq T$, and $\tau_{s,k} = \tau_s + k(T/K)$ for $k = 0, ...K$, $\tau_s = \tau_{s,0}$, $\tau_{s,K} = \tau_s + T$). $P_{i,k}$ are symmetric matrices of compatible dimensions, where K is an integer that may be chosen a priori.

This Lyapunov function allows to solve the conditions at switching instants easier and to deal with uncertainties or LPV systems. Throughout this paper, this Lyapunov function will be used.

3. PROBLEM FORMULATION

Consider the following single input continuous time linear invariant system:

$$\dot{x}(t) = Ax(t) + B_2 u(t) + B_1 w(t) \quad (3)$$
$$y(t) = Cx(t)$$

where $x(t) \in R^n, u(t) \in R, w(t) \in R^p$ and $y(t) \in R^m$ are the state, the control input, the input disturbance and the measured output vectors, respectively. Matrices A, B_2, B_1, C are real matrices of appropriate dimensions. Assume that the pair (A, C) is observable. Now, assume that the system (3) is in the faulty actuator

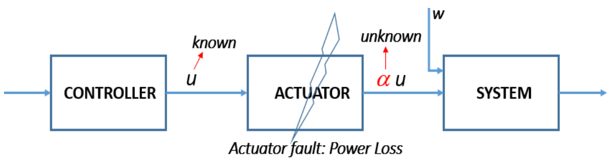

Fig. 1. System with actuator fault

situation, e.g loss of actuator power (Fig.1). Let us model this failure by a multiplicative fault. In fact, denoting $\bar{u}$ is the output of faulty actuator, then:

$$\bar{u}(t) = \alpha u(t) \quad (4)$$

where α stands for the faulty coefficient i.e if $\alpha = 1$, the actuator is healthy , and when $\alpha = 0.8$, the actuator loses 20% of its efficiency. Then, the system (6) becomes:

$$\dot{x}(t) = Ax(t) + B_2\alpha u(t) + B_1 w(t) \quad (5)$$
$$y(t) = Cx(t)$$

As mentioned previously, the objective of this work is to estimate the actuator fault. To do this, the main problem consists in estimating the coefficient α, which will be tackled, based on an extended switched observer and presented in the sequel.

Remark : It can be seen that if the control input $u(t) = 0$, then the fault information α in (5) becomes unobservable. The solution in the synthesis step becomes therefore infeasible and the coefficient α cannot be estimated . Thus, in this work, it is assumed that in the synthesis step, the control input $u(t)$ is kept always from zero. To account for the change the sign of $u(t)$, the system will be rewritten as a switched LPV system in the next section.

It is worth noting that the corresponding additive fault $f(t) = (1 - \alpha)u(t)$ is a time variant signal and depends on the value of the control input $u(t)$, which is more complex to handle and estimate.

4. ACTUATOR FAULT ESTIMATION BASED SWITCHED OBSERVER

The actuator fault estimation in this work is done by estimating a effciency coefficient α of the actuator, and is given step by step in the sequel. Let us consider 2 cases studies: system without input disturbance and system with input disturbance.

4.1 System without input disturbance

In this section, it is assumed that the input disturbance $w(t)$ is null or vanishes. Then, from the system (5), one obtains a system with the fault actuator but without input disturbance as follows:

$$\dot{x}(t) = Ax(t) + B_2\alpha u(t) \quad (6)$$
$$y(t) = Cx(t)$$

where α has to be estimated. Since the control input $u(t)$ is known, in this work, one can model the system (6) as an LPV system by choosing $u(t)$ as a time-varying parameter. Moreover, it is assumed that the control input $u(t)$ is bounded, i.e:

$$0 <| u(t) |\leq u_{max}, u_{max} > 0 \quad (7)$$

Let us rewrite $u(t) = |u(t)|sign(u(t))$, and denote $\rho(t) = |u(t)|$ as a time varying parameter. Then,

$$u(t) = \rho(t)sign(u(t)) = \begin{cases} \rho(t) & if \quad u(t)) > 0 \\ -\rho(t) & if \quad u(t) < 0 \end{cases} \quad (8)$$

Note that from (7), $\rho(t)$ is also bounded: $0 < \rho(t) \leq u_{max}$. Moreover, the bounds of ρ shoud be chosen to be able to apply the polytopic approach in the latter. Here, one chooses $\rho(t) \in [\underline{\rho}\ \ \bar{\rho}]$ where $\underline{\rho} = 0.01, \bar{\rho} = u_{max}$.

Then (6) becomes:

$$\dot{x}(t) = Ax(t) + B_2\alpha\rho(t)sign(u(t)) = Ax(t) + B(\rho)\alpha sign(u(t)) \quad (9)$$
$$y(t) = Cx(t)$$

where $B(\rho) = B_2\rho(t)$.

As seen before, α is constant, then $\dot{\alpha} = 0$. Hence the LPV system (9) can be augmented and rewritten in the following form:

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \end{bmatrix} = \underbrace{\begin{bmatrix} A & B(\rho)\sigma \\ 0 & 0 \end{bmatrix}}_{A_e(\rho)} \begin{bmatrix} x \\ \alpha \end{bmatrix} \quad (10)$$

$$y = \underbrace{[C \ \ 0]}_{C_e} \begin{bmatrix} x \\ \alpha \end{bmatrix}$$

where $\sigma = sign(u(t))$

Furthermore, it can be rewritten in a switched LPV system form (Lu and Wu (2004)) as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \end{bmatrix} = A_{e,\sigma}(\rho) \begin{bmatrix} x \\ \alpha \end{bmatrix} \tag{11}$$

$$y = \underbrace{[C \ \ 0]}_{C_e} \begin{bmatrix} x \\ \alpha \end{bmatrix}$$

where $A_{e,\sigma}(\rho)$ is switched between two modes $A_{e,1}(\rho), A_{e,2}(\rho)$ according to the switched signal $\sigma(t) = sign(u(t))$ as follows:

$$A_{e,\sigma}(\rho) = \begin{cases} \underbrace{\begin{bmatrix} A & B(\rho) \\ 0 & 0 \end{bmatrix}}_{A_{e,1}(\rho)} & if \quad u(t) > 0 \\ \underbrace{\begin{bmatrix} A & -B(\rho) \\ 0 & 0 \end{bmatrix}}_{A_{e,2}(\rho)} & if \quad u(t) < 0 \end{cases} \tag{12}$$

It is worth noting that the previous definition of switched system allows to handle control input signals that crosses the 0 value, which cannot be done when choosing $u(t)$ as scheduling parameter.

Now, the following LPV extended switched observer is used to estimate the extended state of the switched system (11):

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\alpha}} \end{bmatrix} = A_{e,\sigma}(\rho) \begin{bmatrix} \hat{x} \\ \hat{\alpha} \end{bmatrix} + K_\sigma(t)(y - \hat{y}) \tag{13}$$

$$\hat{y} = C_e \begin{bmatrix} \hat{x} \\ \hat{\alpha} \end{bmatrix}$$

From (11) and (13), the estimation error $e(t)$ satisfies:

$$\dot{e} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_\alpha \end{bmatrix} = (A_{e,\sigma}(\rho) - K_\sigma(t) C_e) \begin{bmatrix} e_x \\ e_\alpha \end{bmatrix} \tag{14}$$

The first main result about stability of the estimation error is given below.

Theorem 1. Consider the switched LPV system (11) and the extended switched observer (13), if there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0, ...K, \quad i = 1, ...M$ ($M = 2$: number of subsytems), of appropriate dimensions and K is prescribed integer, such that for all $i = 1, ..M$ and $j = 1, ...N$, ($N = 2$: number of the vertices of the polytope), the following LMIs hold:

$$\frac{(P_{i,k+1} - P_{i,k})}{T/K} + A^{(j)\prime}_{e,\sigma} P_{i,h} - C_e' Y_{i,h}' + P_{i,h} A^{(j)}_{e,\sigma} - Y_{i,h} C_e < 0 \tag{15}$$

for $k = 0, ...K-1$, $h = k, k+1$,

$$A^{(j)\prime}_{e,\sigma} P_{i,K} - C_e' Y_{i,K}' + P_{i,K} A^{(j)}_{e,\sigma} + P_{i,K} A^{(j)}_{e,\sigma} - Y_{i,K} C_e < 0 \tag{16}$$

$$P_{i,K} - P_{l,0} \geq 0 \ \ \forall l = 1, ..i-1, i+1, ...M. \tag{17}$$

then

$$K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1} Y_{\sigma(t)}(t) = \begin{cases} \hat{P}_{i,k}^{-1} \hat{Y}_{i,k} & if \quad t \in [\tau_{s,k}, \ \tau_{s,k+1}) \\ P_{i,K}^{-1} Y_{i,K} & if \quad t \in [\tau_{s,K}, \ \tau_{s+1,0}) \\ P_{i_0,K}^{-1} Y_{i_0,K} & if \quad t \in [0, \tau_1) \end{cases} \tag{18}$$

is the gain of the extended observer (13) and the error estimation asymptotically converges to zero for a given dwell time T. where

$$Y_{\sigma(t)}(t) = \begin{cases} Y_{i,k} + (Y_{i,k+1} - Y_{i,k}) \dfrac{t - \tau_{s,k}}{T/K} := \hat{Y}_{i,k} & if \quad t \in [\tau_{s,k}, \ \tau_{s,k+1}) \\ Y_{i,K} & if \quad t \in [\tau_{s,K}, \ \tau_{s+1,0}) \\ Y_{i_0,K} & if \quad t \in [0, \tau_1) \end{cases} \tag{19}$$

Proof:

Define the Lyapunov function candidate: $V_\sigma(t) = e(t)' P_\sigma(t) e(t)$ where $P_\sigma(t) = P_\sigma(t)' > 0$. Denoting $Y_{\sigma(t)}(t) = P_{\sigma(t)}(t) K_{\sigma(t)}(t)$. Then the estimation error in (14) is asymptotically stable if: $\dot{V}_\sigma < 0 \Leftrightarrow \dot{e}' P_\sigma e + e' P_\sigma \dot{e} + e' \dot{P}_\sigma e < 0 \Leftrightarrow$

$$(A_{e,\sigma}(\rho) - K_\sigma(t) C_e)' P_\sigma(t) + P_\sigma(t)(A_{e,\sigma}(\rho) - K_\sigma(t) C_e) + \dot{P}_\sigma(t) < 0 \tag{20}$$

$$\Leftrightarrow A_{e,\sigma}(\rho)' P_\sigma(t) - C_e' Y_\sigma(t)' + P_\sigma(t) A_{e,\sigma}(\rho) - Y_\sigma(t) C_e + \dot{P}_\sigma(t) < 0 \tag{21}$$

From the formula of $P_\sigma(t)$ in (2), we ensure (21)$\Leftrightarrow$

$$A_{e,\sigma}(\rho)' P_{i,h} - C_e' Y_{i,h}' + P_{i,h} A_{e,\sigma}(\rho) - Y_{i,h} C_e + \frac{(P_{i,k+1} - P_{i,k})}{T/K} < 0 \tag{22}$$

holds for $h = k, k+1$, $i = 1, 2$, $k = 0, ...K-1$.

and

$$A_{e,\sigma}(\rho)' P_{i,K} - C_e' Y_{i,K}' + P_{i,K} A_{e,\sigma}(\rho) + P_{i,K} A_{e,\sigma}(\rho) - Y_{i,K} C_e < 0 \tag{23}$$

hold for $i = 1, 2$.

The equation (22) guarantees that the Lyapunov function $V_\sigma(t)$ decreases during the time interval $t \in [\tau_{s,0}, \tau_{s,K})$. The LMIs (23) ensures that $V_\sigma(t)$ decreases after the dwell time and before the next switching time, i.e $t \in [\tau_{s,K}, \tau_{s+1,0})$.

From the definition of $P_{\sigma(t)}(t)$, consider that at the switching instant τ_k, the system switches from the mode i to the mode l, and $V(\tau_k^-) = x(\tau_k)' P_{i,K} x(\tau_k), V(\tau_k) = x(\tau_k)' P_{l,0} x(\tau_k)$. To guarantee the non-increasing of the Lyapunov function at the switching instants, we must ensure that $V(\tau_k) \leq V(\tau_k^-)$ which corresponds to the inequality (17), i.e:

$$P_{i,K} - P_{l,0} \geq 0 \ \ \forall l = 1, ..i-1, i+1, ...M.$$

In our case, we have 2 subsystems, $M = 2$, i.e $P_{1,1} - P_{2,0} \geq 0$ and $P_{2,1} - P_{1,0} \geq 0$.

Now, in order to resolve the LMIs in (22), (23), that are parameter dependent, the polytopic approach for LPV systems is considred where the 2-vertices polytope is given by $\Omega_\rho = \left[\underline{\rho} \ \bar{\rho}\right]$. The LMIs (22), (23) is therefore solved at the vertices of the polytope (i.e $\underline{\rho}, \bar{\rho}$). Then, one obtains the LMIs in (15), (16). Finally, $K_\sigma(t) = P_\sigma^{-1} Y_\sigma(t)$ is the gain of the switched extended observer (13).□

The following theorem extends the previous one imposing a pole placement constraint for the observer design, imposing a prefixed decay rate for the exponential stability of the estimation error.

Theorem 2. Consider the switched LPV system (11) and the switched extended observer (13), if there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0, ...K, \quad i = 1, ...M$ ($M = 2$: number of subsytems), of appropriate dimensions, K is prescribed integer, and a positive scalar β such that for all $i = 1, ..M$ and $j = 1, ...N$, ($N = 2$: number of the vertices of the polytope), the following LMIs hold:

$$\frac{(P_{i,k+1} - P_{i,k})}{T/K} + A^{(j)\prime}_{e,\sigma} P_{i,h} - C_e' Y_{i,h}' + P_{i,h} A^{(j)}_{e,\sigma} - Y_{i,h} C_e + 2\beta P_{i,h} < 0 \tag{24}$$

for $k = 0, ...K-1$, $h = k, k+1$,

$$A^{(j)\prime}_{e,\sigma} P_{i,K} - C_e' Y_{i,K}' + P_{i,K} A^{(j)}_{e,\sigma} + P_{i,K} A^{(j)}_{e,\sigma} - Y_{i,K} C_e + 2\beta P_{i,K} < 0 \tag{25}$$

$$P_{i,K} - P_{l,0} \geq 0 \ \ \forall l = 1, ..i-1, i+1, ...M. \tag{26}$$

then $K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1} Y_{\sigma(t)}(t)$ is the gain of the extended observer (13) and this observer converges with a decay rate of β, i.e the estimation error $e(t)$ asymptotically exponentially converges to zero for a dwell time of T.

Proof: The idea of the proof is similar to the "Theorem 1" and is omitted here. We need only to note that in order to increase the convergence's speed of the estimation, one introduces another term of *'decay rate'*, i.e, if there exists a positive scalar β such that $\dot{V}_\sigma(t) \leq -2\beta V_\sigma(t)$, then the estimation error converges to zero with a decay rate of β. □

4.2 System with input disturbance

In this section, the fault estimation in the presence of input disturbance is considered, i.e, the system (5) is taken into account:

$$\dot{x}(t) = Ax(t) + B_2\alpha u(t) + B_1 w(t) \quad (27)$$
$$y(t) = Cx(t)$$

Similarly, one has $u(t) = |u(t)|sign(u(t))$, and denote $\rho(t) = |u(t)|$ as a time varying parameter and $\rho(t)$ is also bounded: $\underline{\rho} \le \rho(t) \le \bar{\rho}$, where $\underline{\rho} = 0.01, \bar{\rho} = u_{max}$.
Then, (27) becomes:

$$\dot{x}(t) = Ax(t) + B(\rho)\alpha sign(u(t)) + B_1 w(t) \quad (28)$$
$$y(t) = Cx(t)$$

The system (28) is augemented as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \end{bmatrix} = A_{e,\sigma}(\rho)\begin{bmatrix} x \\ \alpha \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w \quad (29)$$
$$y = \underbrace{[C \quad 0]}_{C_e}\begin{bmatrix} x \\ \alpha \end{bmatrix}$$

where $A_{e,\sigma}(\rho)$ is switched between two modes $A_{e,1}(\rho), A_{e,2}(\rho)$ according to the signal $\sigma(t) = sign(u(t))$, as in (12).
Then, the following switched extended observer is proposed to estimate the system's state and the coefficient α :

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\alpha}} \end{bmatrix} = A_{e,\sigma}(\rho)\begin{bmatrix} \hat{x} \\ \hat{\alpha} \end{bmatrix} + K_\sigma(t)(y - \hat{y}) \quad (30)$$
$$\hat{y} = C_e\begin{bmatrix} \hat{x} \\ \hat{\alpha} \end{bmatrix}$$

From (29) and (30), the estimation error $e(t)$ satisfies:

$$\dot{e} = \begin{bmatrix} \dot{e_x} \\ \dot{e}_\alpha \end{bmatrix} = (A_{e,\sigma}(\rho) - K_\sigma(t)C_e)\begin{bmatrix} e_x \\ e_\alpha \end{bmatrix} + B_{1e}w \quad (31)$$

where $B_{1e} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$ and $K_\sigma(t)$ is the observer gain which has to be determined.
The extended observer (30) and the estimation error (31) are affected by the disturbance effect $w(t)$. The observer design problem in this case involves the calculation of the observer gain $K_\sigma(t)$ such that:

- when $w(t) \equiv 0$, the observer is asymptotically stable
- when $w(t) \neq 0$, the following L_2-induced gain performance criterion is satisfied :

$$min \;\; \gamma \;\; s.t \;\; \| e \|_2 < \gamma \| w \|_2 \quad (32)$$

where $\| . \|_2$ stands for L_2 norm.
The following theorem solves the above problem.

Theorem 3. Consider the switched system (29) and the switched extended observer (30), if there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0,...K, \quad i = 1,...M$ ($M = 2$: number of subsytems), of appropriate dimensions and K is prescribed integer, such that for all $i = 1,..M$ and $j = 1,...N$, ($N = 2$: number of the vertices of the polytope), the following LMIs hold:

$$\begin{bmatrix} \frac{(P_{i,k+1} - P_{i,k})}{T/K} + A^{(j)\prime}_{e,\sigma}P_{i,h} - C_e'Y_{i,h}' + P_{i,h}A^{(j)}_{e,\sigma} - Y_{i,h}C_e & P_{i,h}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (33)$$

for $k = 0,...K-1$, $h = k, k+1$,

$$\begin{bmatrix} A^{(j)\prime}_{e,\sigma}P_{i,K} - C_e'Y_{i,K}' + P_{i,K}A^{(j)}_{e,\sigma} + P_{i,K}A^{(j)}_{e,\sigma} - Y_{i,K}C_e & P_{i,K}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (34)$$

$$P_{i,K} - P_{l,0} \ge 0 \;\; \forall l = 1,..i-1, i+1,...M. \quad (35)$$

then

$$K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1}Y_{\sigma(t)}(t) = \begin{cases} \hat{P}_{i,k}^{-1}\hat{Y}_{i,k} & if \quad t \in [\tau_{s,k}, \; \tau_{s,k+1}) \\ P_{i,K}^{-1}Y_{i,K} & if \quad t \in [\tau_{s,K}, \; \tau_{s+1,0}) \\ P_{i_0,K}^{-1}Y_{i_0,K} & if \quad t \in [0, \tau_1) \end{cases} \quad (36)$$

is the gain of the extended observer (30) and the error estimation asymptotically converges to zero for a dwell time of T. where

$$Y_{\sigma(t)}(t) = \begin{cases} Y_{i,k} + (Y_{i,k+1} - Y_{i,k})\frac{t - \tau_{s,k}}{T/K} := \hat{Y}_{i,k} & if \quad t \in [\tau_{s,k}, \; \tau_{s,k+1}) \\ Y_{i,K} & if \quad t \in [\tau_{s,K}, \; \tau_{s+1,0}) \\ Y_{i_0,K} & if \quad t \in [0, \tau_1) \end{cases} \quad (37)$$

Proof: From the Bounded Real Lemma, the condition (32) is satisfied if the following condition holds:

$$\dot{V}(t) + e'e - \gamma^2 w'w < 0 \quad (38)$$

$\Leftrightarrow$

$$\begin{bmatrix} (A_{e,\sigma}(\rho) - K_\sigma(t)C_e)'P_\sigma(t) + P_\sigma(t)(A_{e,\sigma}(\rho) - K_\sigma(t)C_e) + \dot{P}_\sigma(t) & P_\sigma(t)B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (39)$$

From the formula of $P_\sigma(t)$ in (2), (39) is satisfied if

$$\begin{bmatrix} A_{e,\sigma}(\rho)'P_{i,h} - C_e'Y_{i,h}' + P_{i,h}A_{e,\sigma}(\rho) - Y_{i,h}C_e + \frac{(P_{i,k+1} - P_{i,k})}{T/K} & P_{i,h}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (40)$$

holds for $h = k, k+1$, $i = 1,2$, $k = 0,...K-1$.
and

$$\begin{bmatrix} A_{e,\sigma}(\rho)'P_{i,K} - C_e'Y_{i,K}' + P_{i,K}A_{e,\sigma}(\rho) + P_{i,K}A_{e,\sigma}(\rho) - Y_{i,K}C_e & P_{i,K}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (41)$$

hold for $i = 1,2$.
The equation (40) guarantees that the Lyapunov function $V_\sigma(t)$ decreases and (38) holds during the time intervals i.e $t \in [\tau_{s,0}, \tau_{s,K})$. The LMIs (41) ensure that $V_\sigma(t)$ decreases and that (38) holds after the dwell time and before the next switching instant, i.e $t \in [\tau_{s,K}, \tau_{s+1,0})$.
From the definition of $P_\sigma(t)$, consider that at instant τ_k, the system switches from the mode i to the mode l, to guarantee the non-increasing of the Lyapunov function at the switching instants, we must ensure:

$$P_{i,K} - P_{l,0} \ge 0 \;\; \forall l = 1,..i-1, i+1,...M. \quad (42)$$

In our case, we have 2 subsystems, $M = 2$, i.e $P_{1,1} - P_{2,0} \ge 0$ and $P_{2,1} - P_{1,0} \ge 0$.
Now, in order to resolve the LMIs in (40), (41), we apply the polytopic solution for LPV system where the polytope is given by $\Omega_\rho = \left[\underline{\rho} \;\; \bar{\rho}\right]$ and obtain the LMIs in (33), (34). ■
The next result extends the previosu one imposing a prefixed decay rate on the convergence of the estimation error.

Theorem 4. Consider the switched system (29) and the switched observer (30), if there exists a collection of matrices $P_{i,k} > 0, Y_{i,k}, k = 0,...K, \quad i = 1,...M$ ($M = 2$: number of subsytems), of appropriate dimensions, K is prescribed integer, and a positive scalar β such that for all $i = 1,..M$ and $j = 1,...N$, ($N = 2$: number of the vertices of the polytope), the following LMIs hold:

$$\begin{bmatrix} \frac{(P_{i,k+1}-P_{i,k})}{T/K} + A^{(j)'}_{e,\sigma}P_{i,h} - C_e'Y_{i,h}' + P_{i,h}A^{(j)}_{e,\sigma} - Y_{i,h}C_e + 2\beta P_{i,h} & P_{i,h}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{43}$$

for $k = 0, ...K-1$, $h = k, k+1$,

$$\begin{bmatrix} A^{(j)'}_{e,\sigma}P_{i,K} - C_e'Y_{i,K}' + P_{i,K}A^{(j)}_{e,\sigma} + P_{i,K}A^{(j)}_{e,\sigma} - Y_{i,K}C_e + 2\beta P_{i,K} & P_{i,K}B_{1e} & I \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{44}$$

$$P_{i,K} - P_{l,0} \geq 0 \ \ \forall l = 1, ..i-1, i+1, ...M. \tag{45}$$

then $K_{\sigma(t)}(t) = P_{\sigma(t)}(t)^{-1}Y_{\sigma(t)}(t)$ is the gain of the extended observer (30) and the error estimation asymptotically converges to zero for a dwell time of T.

Proof: The proof is similar to the last cases and is omitted here for the simplification.

Finally, in order to design the switched observer (30), one has to solve the following optimization problem:

$$\min_{P_{i,k}, Y_{i,k}} \gamma^2 \quad \text{subject to} \ \ (43),(44),(45) \ \ \textit{and} \ \ P_{i,k} > 0 \tag{46}$$

By solving this optimization problem, one can derive $P_\sigma(t), Y_\sigma(t)$ and the observer gain is calculated by $K_\sigma(t) = P_\sigma(t)^{-1}Y_\sigma(t)$.

5. NUMERICAL EXEMPLE

The following numerical example illustrates the effectiveness of the proposed observer. Let consider a LTI system of the form (3) with the following matrices:

$$A = \begin{bmatrix} -3 & -1 \\ 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}$$

The system is subject to the control input $u(t) = 1 + 2\sin(t)$ (see Fig. 2) and the initial value $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}'$. Thus, one has : $-1 \leq u(t) \leq 3$ and $0 < |u(t)| \leq 3$.

Then, the time varying parameter $\rho(t) = |u(t)|$ is assumed to be bounded by: $0.01 \leq \rho \leq 3$ and $A_{e,\sigma}(\rho)$ is switched between two modes $A_{e,1}(\rho), A_{e,2}(\rho)$ according to the switched signal $\sigma(t) = sign(u(t))$, and is given by (12), and at each mode, one has 2 matrices corresponding to 2 vertices of the polytope $\Omega_\rho = \begin{bmatrix} \underline{\rho} & \bar{\rho} \end{bmatrix}$ as follows:

$$\begin{cases} A_{e,1}(\rho) = \left\{ \begin{bmatrix} -3 & -2 & 0.01 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -3 & -2 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \\ A_{e,2}(\rho) = \left\{ \begin{bmatrix} -3 & -2 & -0.01 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -3 & -2 & -3 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \end{cases}$$

The figure Fig. 2 shows also the scheduling parmeter and the switched signal.

Finamly, as seen in figure 3 the efficiency coefficient of the actuator is such that, for $t \in [0-5]s$, $\alpha = 1$, the actuator is healthy, then, for $t > 5s$, $\alpha = 0.5$, the actuator loses 50 % of its efficiency.

Now, the different results are given for the two case studies (without and with input disturbance) with the observer design of section 3.

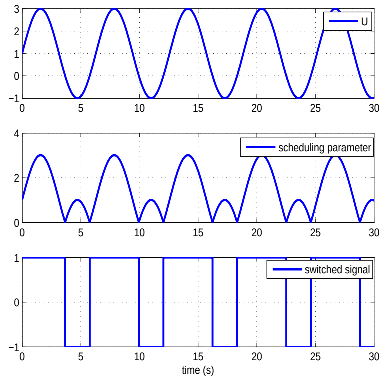

Fig. 2. The control input, varying parameter and switched signal

5.1 *System without input disturbance (case 1)*

In this section, the fault estimation for the system without input disturbance is considered.

By solving the LMIs (24,25, 26) (where the decay rate is taken into account) for the 2 modes $A_{e,1}(\rho), A_{e,2}(\rho)$ to obtain the matrices $P_{i,k}, Y_{i,k}$, then the observer gain is calculated by $K_\sigma(t) = P_\sigma(t)^{-1}Y_\sigma(t)$.

Fig.3 and Fig.4 show the estimation of the faulty coefficient α and the system states, with and without taking into account the decay rate in the observer design step. It shows that the switched observer allows to estimate the faulty coefficient as well as the system state. Moreover adding a decay rate constraint allows to get a faster response of the observer.

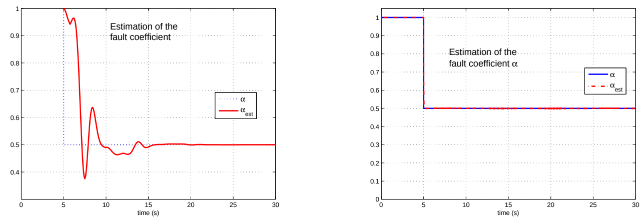

Fig. 3. Estimation of α without decay rate $\beta = 0$ (left) and with $\beta = 1$ (right)- (case 1)

5.2 *System with input disturbance (case 2)*

The disturbance effects on the estimation are considered in this section. See the disturbance appearing at $t = 10s$ in the Fig. 5.

By solving the LMIs (43,44, 45) (where the decay rate is taken into account) for the 2 modes $A_{e,1}(\rho), A_{e,2}(\rho)$ to obtain the matrices $P_{i,k}, Y_{i,k}$, then the observer gain is calculated by $K_\sigma(t) = P_\sigma(t)^{-1}Y_\sigma(t)$.

Fig. 6, Fig. 7, show the estimation of the effciency coefficient α and the system states without and with taking into account the decay rate. Obviously, despite of the input disturbance $w(t)$, the switched observer allows to have a good estimation of the coefficient α and the system states, in particular for the observer with decay rate which gives a much better result.

6. CONCLUSION

In this paper, an actuator fault estimation was proposed within the LPV approach. The actuator fault is modeled in a multiplicative way by using a constant coefficient ($\alpha \in [0 \ \ 1]$) that represents the fault information. This allows to facilitate the fault

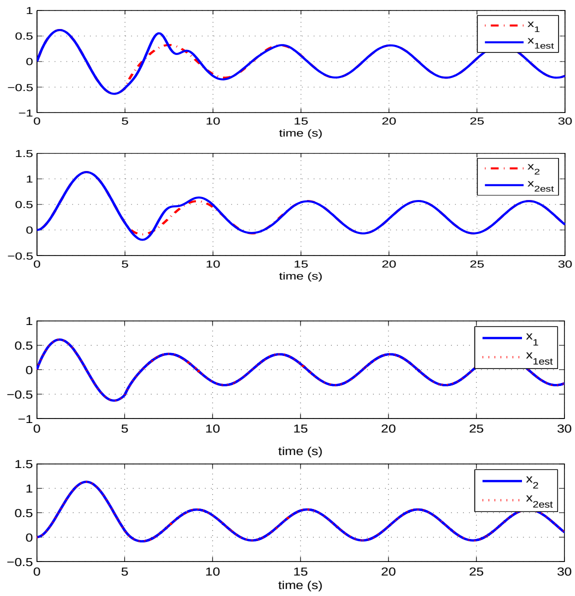

Fig. 4. State estimation without decay rate $\beta = 0$ (left) and with $\beta = 1$ (right)- (case 1)

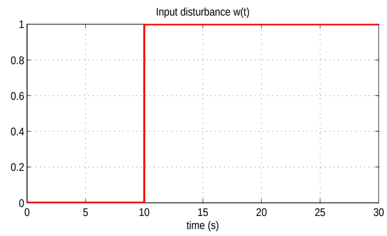

Fig. 5. Input disturbance (case 2)

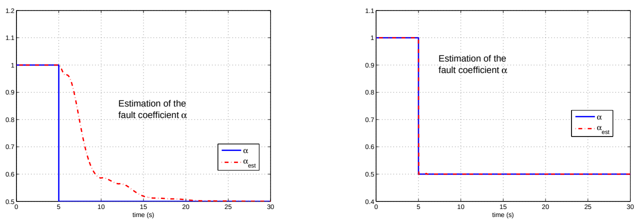

Fig. 6. Estimation of α without decay rate $\beta = 0$ (left) and with $\beta = 1$ (right)- (case 2)

estimation. The estimation is based on an extended switched observer and the simulation results show the effectiveness of the proposed approach. In the future work, this approach can be applied on a real system such as the suspension system. Therein, we want to estimate the fault on the damper malfunction (e.g. oil leakage of the damper,...) where some preliminary results are presented in Nguyen et al. (2015). Moreover, some fault tolerant control strategies can be developped, based on the proposed fault diagnosis.

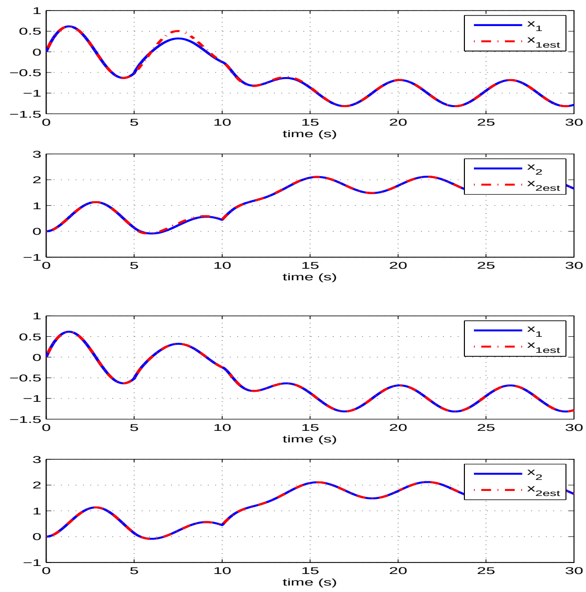

Fig. 7. State estimation without decay rate $\beta = 0$ (left) and with $\beta = 1$ (right)- (case 2)

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